Functors

Or: why your abstractions are weak

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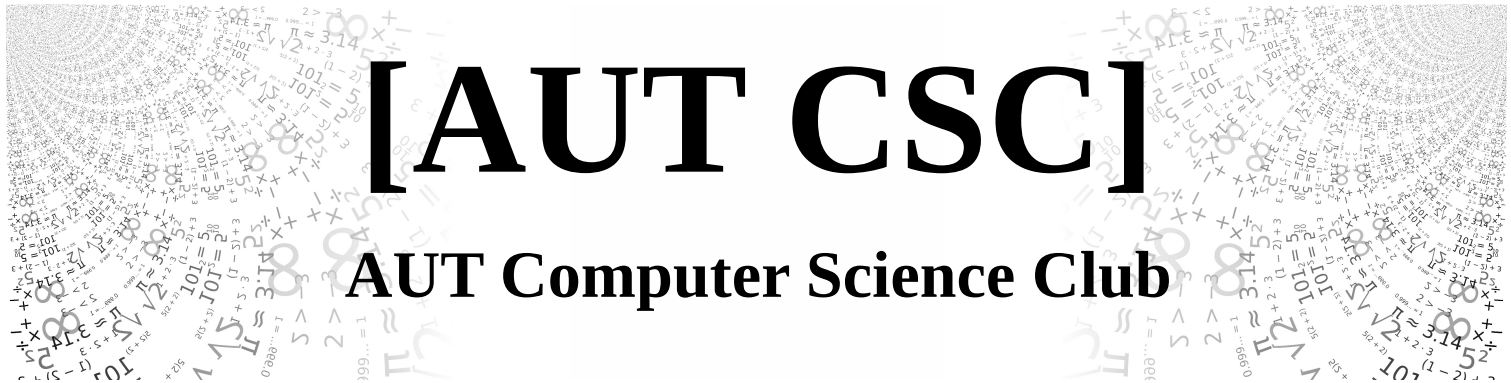

Outline

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These things are very different, aren't they? Your programming languages sure seem to think so...

The truth

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Yes, your languages have been *lying* to you.

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Let's fix this, shall we?

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plus :: Int -> Int -> Int -- functions have types too!
plus x y = x + y -- and definitions
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We will typically use `a`,`b` for type parameters.

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This seems rather strange — why not have *pure* code be the default, and then use the type system to *state* what other behaviour we want to have?

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It seems all we can do is re-write every single thing to handle every single condition possible, which is ridiculous and impossible. We deserve better from our languages!

What you're probably thinking now

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We call this the *functor law*.

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We can define (correct) functor instances for `Maybe`, `Either`, `[]`, `(,)`, `IO`, most container types,...

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Furthermore, we can define (correct) functors for *all* the types which describe all the effects we mentioned earlier. Now, we can treat them all the same, just by writing functions against `Functor` instead. What's more, this requires *no special support from the language* — if we can define the functor, we can do this!

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=> Just (id x) == Just x
=> Just x == Just x
```

Proving the functor law for Maybe

Lemma

Our definition of `Maybe` *follows the functor law.*

Proof.

We must show that, for our definition of `Maybe`, we have:

```
map id == id
```

If `Maybe a == Nothing`, we have:

```
map id Nothing == id Nothing
=> Nothing == Nothing
```

Otherwise, `Maybe a == Just x`, and we have:

```
map id (Just x) == id (Just x)
=> Just (id x) == Just x
=> Just x == Just x
```

Thus, our definition of `Maybe` follows the functor law. □

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Luckily for us, there are other, more powerful abstractions built on the functor which solve *both* of these problems.

Questions?