

Building up

Questions

Or: How computer science is like cooking

March 30, 2017

What are algorithms,
and why do we care?

Building up

Going asymptotic

Questions

Overview

What are algorithms, and why do we care?

Building up

Going asymptotic

Questions

**What are algorithms,
and why do we care?**

- Definition of an algorithm
- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

**What are algorithms, and why do
we care?**

Definition of an algorithm

What are algorithms,
and why do we care?

- Definition of an algorithm

- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

“A procedure, described in a finite number of instructions, for transforming inputs into an output to solve a specific problem. The instructions must be unambiguous, guaranteed to terminate, and solve the problem correctly in all cases.”

Definition of an algorithm

What are algorithms,
and why do we care?

- Definition of an algorithm

- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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Alternatively, you can think of an algorithm as a *recipe*:

Definition of an algorithm

What are algorithms,
and why do we care?

- Definition of an algorithm

- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

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Definition of an algorithm

What are algorithms,
and why do we care?

- Definition of an algorithm

- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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Definition of an algorithm

What are algorithms,
and why do we care?

- Definition of an algorithm

- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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- The ingredients are the *inputs*
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- The (hopefully!) resulting food is the *output*

What are algorithms,
and why do we care?

- Definition of an algorithm

- **Why we care**

- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

Why we care

- Algorithms are *very* fundamental to computer science (which is why it's all about cooking!)

What are algorithms,
and why do we care?

- Definition of an algorithm

- **Why we care**

- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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- Algorithms are *very* fundamental to computer science (which is why it's all about cooking!)
- They define what we can do *at all*, and what we can do *efficiently*

What are algorithms,
and why do we care?

- Definition of an algorithm

- **Why we care**

- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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What are algorithms,
and why do we care?

- Definition of an algorithm

- **Why we care**

- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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What are algorithms,
and why do we care?

- Definition of an algorithm

- **Why we care**

- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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- If you don't know this stuff, everything you do on a computer will run like hot garbage, and you won't know *why*, or what to do about it

What are algorithms,
and why do we care?

- Definition of an algorithm

- **Why we care**

- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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Thus, we really need to be able to *compare* algorithms to each other, and also *understand* why they run the way they do, if we have any hope of making them do our bidding.

What do we mean by 'efficient'?

What are algorithms,
and why do we care?

- Definition of an algorithm
- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

Efficiency is ultimately doing more with less effort. We can measure the 'effort' required to run an algorithm in two ways:

What do we mean by 'efficient'?

What are algorithms,
and why do we care?

- Definition of an algorithm
- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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What do we mean by 'efficient'?

What are algorithms,
and why do we care?

- Definition of an algorithm
- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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What do we mean by 'efficient'?

What are algorithms,
and why do we care?

- Definition of an algorithm
- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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Determining these two factors is an important part of *algorithm analysis*.

What do we mean by 'efficient'?

What are algorithms,
and why do we care?

- Definition of an algorithm
- Why we care
- What do we mean by 'efficient'?

Building up

Going asymptotic

Questions

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Determining these two factors is an important part of *algorithm analysis*. Additionally, any method we use must explain *why* we get the performance it claims.

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

Building up

Analysis by implementation

What are algorithms,
and why do we care?

Building up

- Analysis by
implementation

- A model
- Time and space
complexity
- An example algorithm

Going asymptotic

Questions

“To analyze the efficiency of an algorithm, implement it, run it on some inputs, and measure the time required and the memory used.”

Analysis by implementation

What are algorithms,
and why do we care?

Building up

- Analysis by
implementation

- A model
- Time and space
complexity
- An example algorithm

Going asymptotic

Questions

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Analysis by implementation

What are algorithms,
and why do we care?

Building up

- Analysis by
implementation

- A model
- Time and space
complexity
- An example algorithm

Going asymptotic

Questions

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Analysis by implementation

What are algorithms,
and why do we care?

Building up

- Analysis by
implementation

- A model
- Time and space
complexity
- An example algorithm

Going asymptotic

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What are algorithms,
and why do we care?

Building up

- Analysis by
implementation

- A model
- Time and space
complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

● Analysis by
implementation

- A model
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

● Analysis by
implementation

- A model
- Time and space complexity
- An example algorithm

Going asymptotic

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While this method is useful at the *end* of the process, it's not a very good analytical tool in general. To do better than this, we need a different view of the problem, as well as a different method.

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

A model

In sciences (of all sorts), a *model* is a way of looking at the world which:

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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- *Explains* why phenomena occur like they do.

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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Examples of models include:

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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Examples of models include:

- The Standard Model

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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Models allow us to *analyze* and *understand* their subject matter without having to 'manually verify' everything.

A model of a computer

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

Because algorithms ultimately run on computers, our model must be of a computer.

A model of a computer

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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There are *many* choices of model, depending on what kind of computer we want to study, as well as what aspects of it interest us.

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

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The most traditional (and foundational) model for algorithm analysis is the *random access model* (RAM).

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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The most traditional (and foundational) model for algorithm analysis is the *random access model* (RAM). This closely represents a computer at the time when algorithm analysis first became a topic in its own right (around 1950).

The random access model

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

In RAM, we have access to:

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

The random access model

In RAM, we have access to:

- A single, sequential *processing unit* (PU)

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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In RAM, we have access to:

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- **A model**
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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Any primitive instruction requires one *time unit* to execute.

Time and space complexity

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

Let A be an algorithm, and n be the size of the largest input to A .

Time and space complexity

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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We define the *time complexity of A for inputs of size n* (written $T_A(n)$) as the number of primitive instructions that the PU must execute to produce correct output for A from any inputs of largest size n .

Time and space complexity

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

Time and space complexity

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- An example algorithm

Going asymptotic

Questions

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What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- **An example algorithm**

Going asymptotic

Questions

An example algorithm

Problem: *Given a non-empty array arr of one-word integers, find the largest integer in arr.*

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- **An example algorithm**

Going asymptotic

Questions

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Problem: *Given a non-empty array arr of one-word integers, find the largest integer in arr .*

```
function Max(arr)
   $x \leftarrow arr[1]$ 
  for  $i \in 2, 3, \dots, \text{length}(arr)$  do
    if  $x < arr[i]$  then
       $x \leftarrow arr[i]$ 
    end if
  end for
  return  $x$ 
end function
```

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- **An example algorithm**

Going asymptotic

Questions

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  for i ∈ 2, 3, ..., length(arr) do
    if x < arr[i] then
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$T_{\text{Max}}(n) =$

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- **An example algorithm**

Going asymptotic

Questions

An example algorithm

Problem: *Given a non-empty array arr of one-word integers, find the largest integer in arr.*

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function Max(arr)
    x ← arr[1]
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$$T_{\text{Max}}(n) = 4 + 6(n - 1) + 1 = 6n - 1$$

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What are algorithms,
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Building up

Going asymptotic

- Asymptotic analysis
- Reviewing the example
- Benefits of asymptotic complexity

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Asymptotic analysis

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Building up

Going asymptotic

- Asymptotic analysis
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Ultimately, we don't actually care about what $T(n)$, $S(n)$ *are*.

Asymptotic analysis

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Building up

Going asymptotic

- Asymptotic analysis
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Ultimately, we don't actually care about what $T(n)$, $S(n)$ *are*. What really matters is *how they behave as their input sizes grow*.

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Building up

Going asymptotic

- Asymptotic analysis
- Reviewing the example
- Benefits of asymptotic complexity

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What are algorithms,
and why do we care?

Building up

Going asymptotic

- Asymptotic analysis
- Reviewing the example
- Benefits of asymptotic complexity

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Building up

Going asymptotic

- Asymptotic analysis
- Reviewing the example
- Benefits of asymptotic complexity

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What are algorithms,
and why do we care?

Building up

Going asymptotic

- Asymptotic analysis
- Reviewing the example
- Benefits of asymptotic complexity

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Building up

Going asymptotic

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- Reviewing the example
- Benefits of asymptotic complexity

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Building up

Going asymptotic

- Asymptotic analysis
- Reviewing the example
- Benefits of asymptotic complexity

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Building up

Going asymptotic

- Asymptotic analysis
- Reviewing the example
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We call this kind of analysis *asymptotic*. This is because the input can grow as big as we like, and we're only interested in how the running time (or space) changes with the growth of the input.

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and why do we care?

Building up

Going asymptotic

- Asymptotic analysis
- **Reviewing the example**
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Questions

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Building up

Going asymptotic

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Building up

Going asymptotic

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- Benefits of asymptotic complexity

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Building up

Going asymptotic

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$$T_{\text{Max}}(n) = \cancel{6n} - \cancel{1} \quad \text{WRONG!}$$

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Reviewing the example

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Building up

Going asymptotic

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$T_{\text{Max}}(n)$ **is** $O(n)$

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Building up

Going asymptotic

- Asymptotic analysis
- **Reviewing the example**
- Benefits of asymptotic complexity

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We also say “the time complexity of Max is $O(n)$ ”, or even “Max is $O(n)$ ”.

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Building up

Going asymptotic

- Asymptotic analysis
- **Reviewing the example**
- Benefits of asymptotic complexity

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Benefits of asymptotic complexity

What are algorithms,
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Building up

Going asymptotic

- Asymptotic analysis
- Reviewing the example
- Benefits of asymptotic complexity

Questions

- *Describes* performance on the basis of how our time and space requirements will grow relative the size of the problem, not pegged to a particular machine (or even model!)

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What are algorithms,
and why do we care?

Building up

Going asymptotic

- Asymptotic analysis
- Reviewing the example
- **Benefits of asymptotic complexity**

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- *Describes* performance on the basis of how our time and space requirements will grow relative the size of the problem, not pegged to a particular machine (or even model!)
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and why do we care?

Building up

Going asymptotic

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- Reviewing the example
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Building up

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It's definitely not perfect though — but that's material for another talk.

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