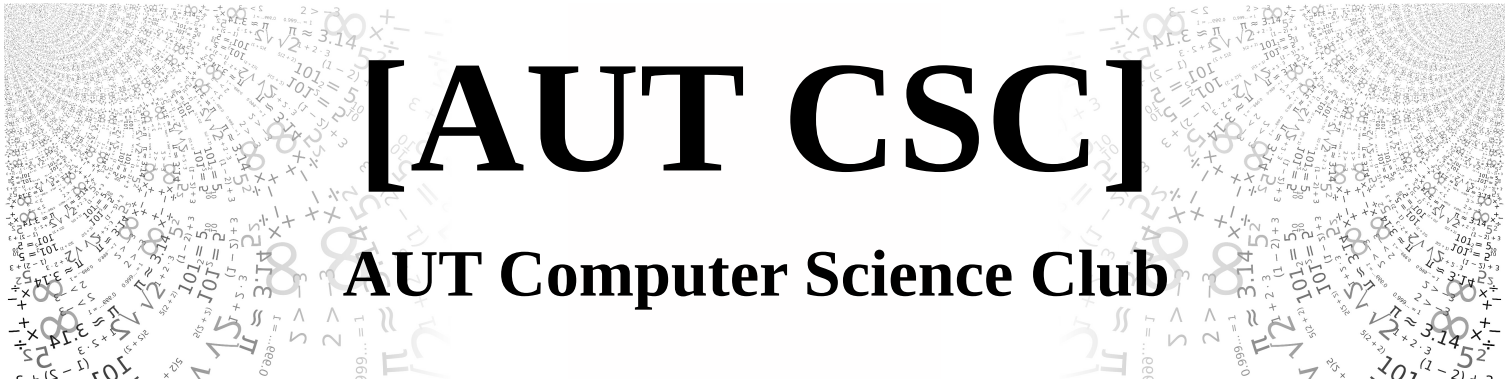
CONS cells

Or: start simple!

Glen Osborne

3rd August, 2017



Outline

All those data structures...

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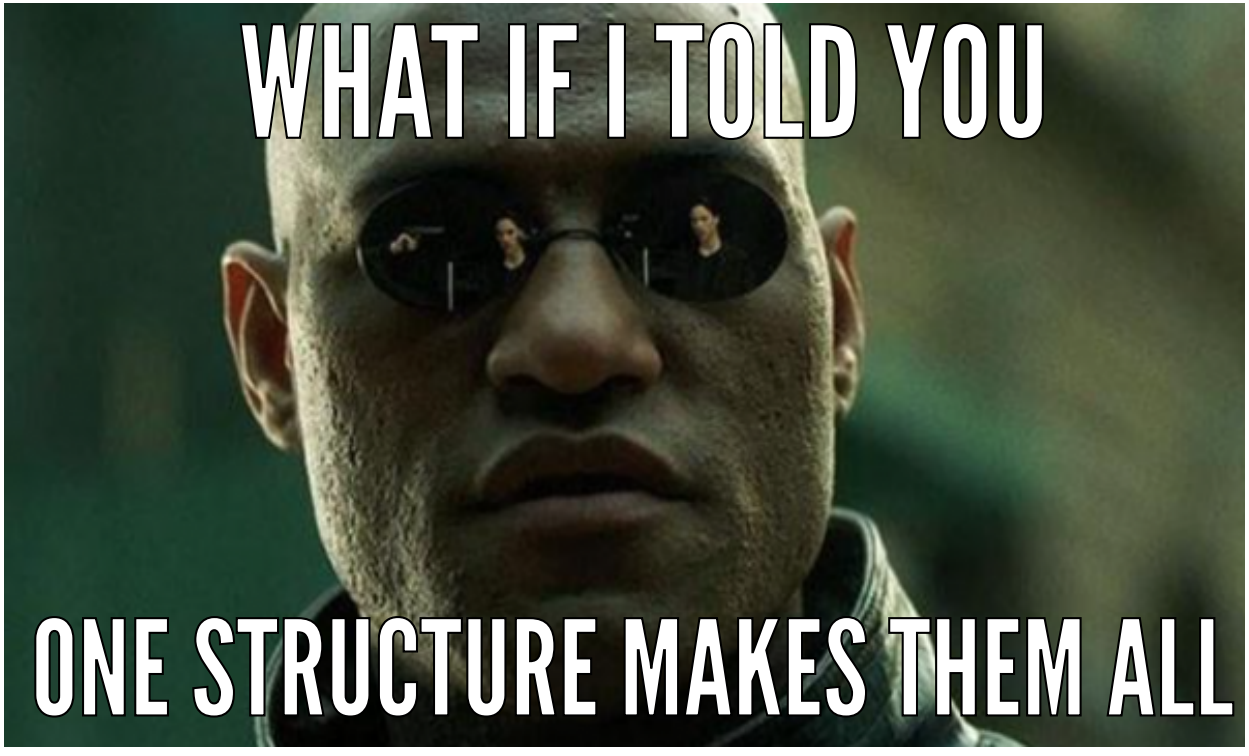
These can be *very* complicated to understand and implement! They also often 'lock you in' to a fixed set of operations — and whether these are the ones you need can be hard to determine when initially solving a problem.

A ‘novel’ proposition



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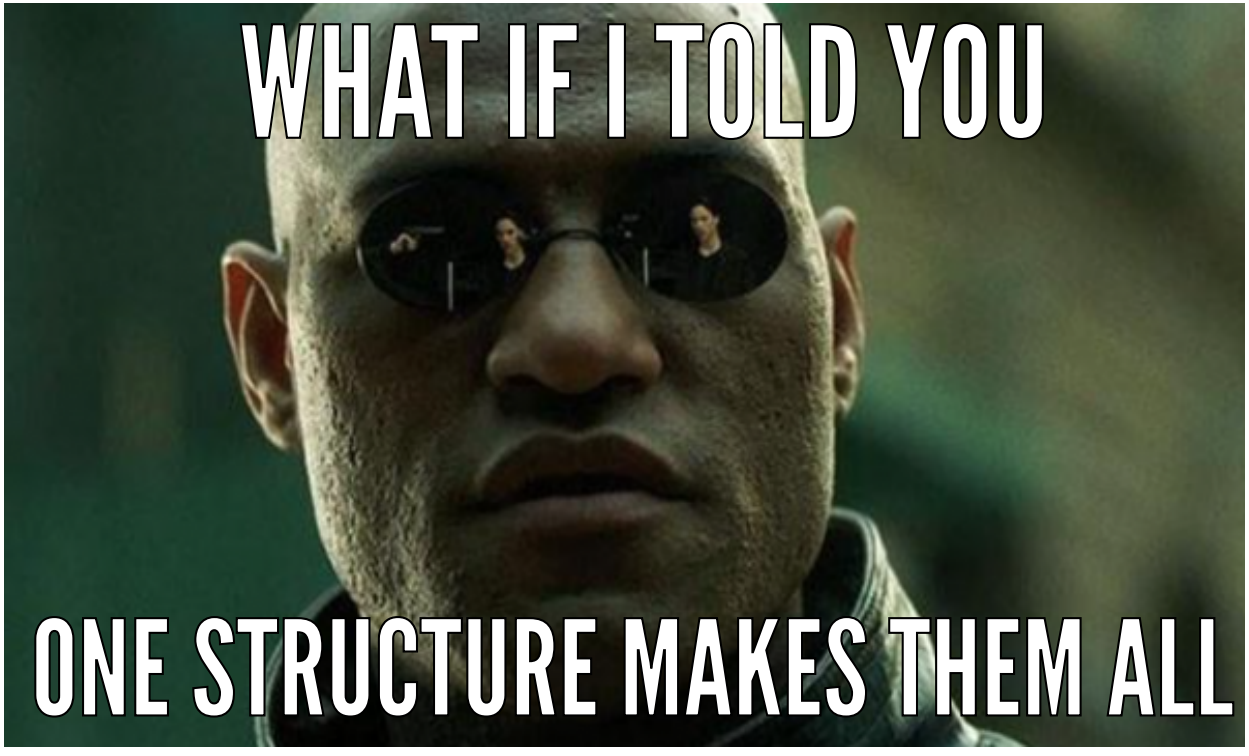

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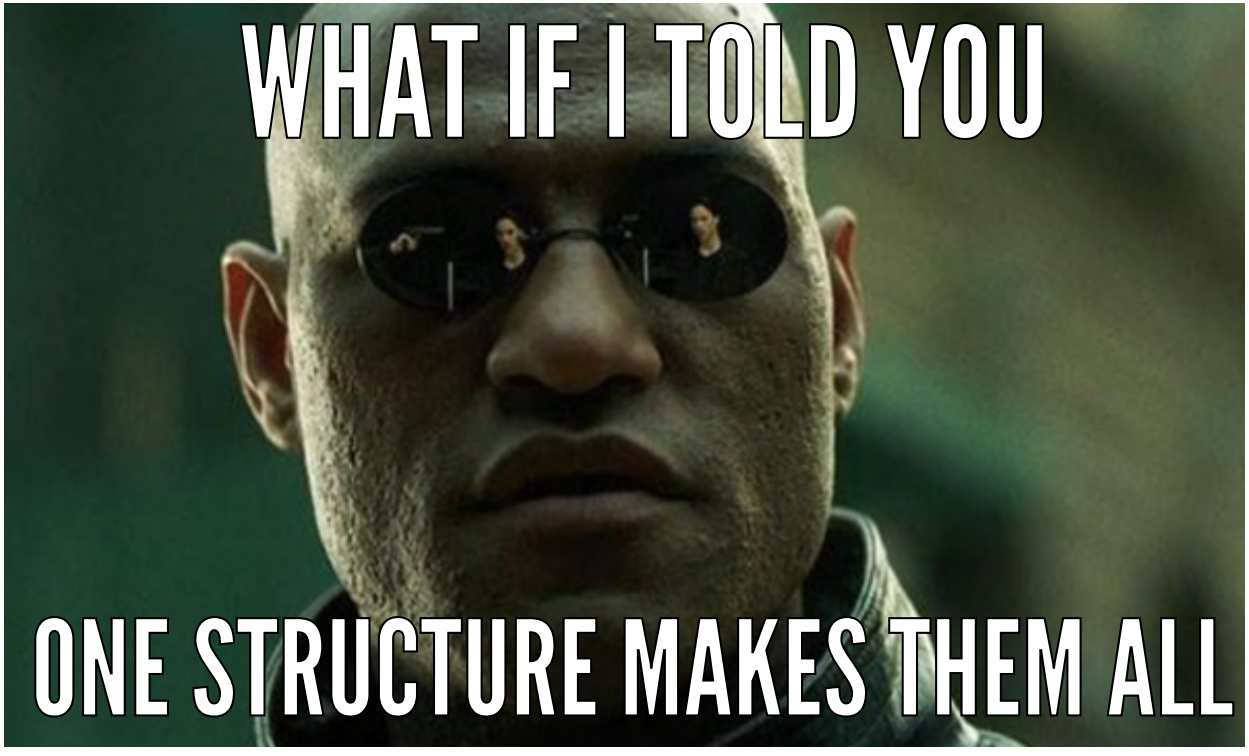

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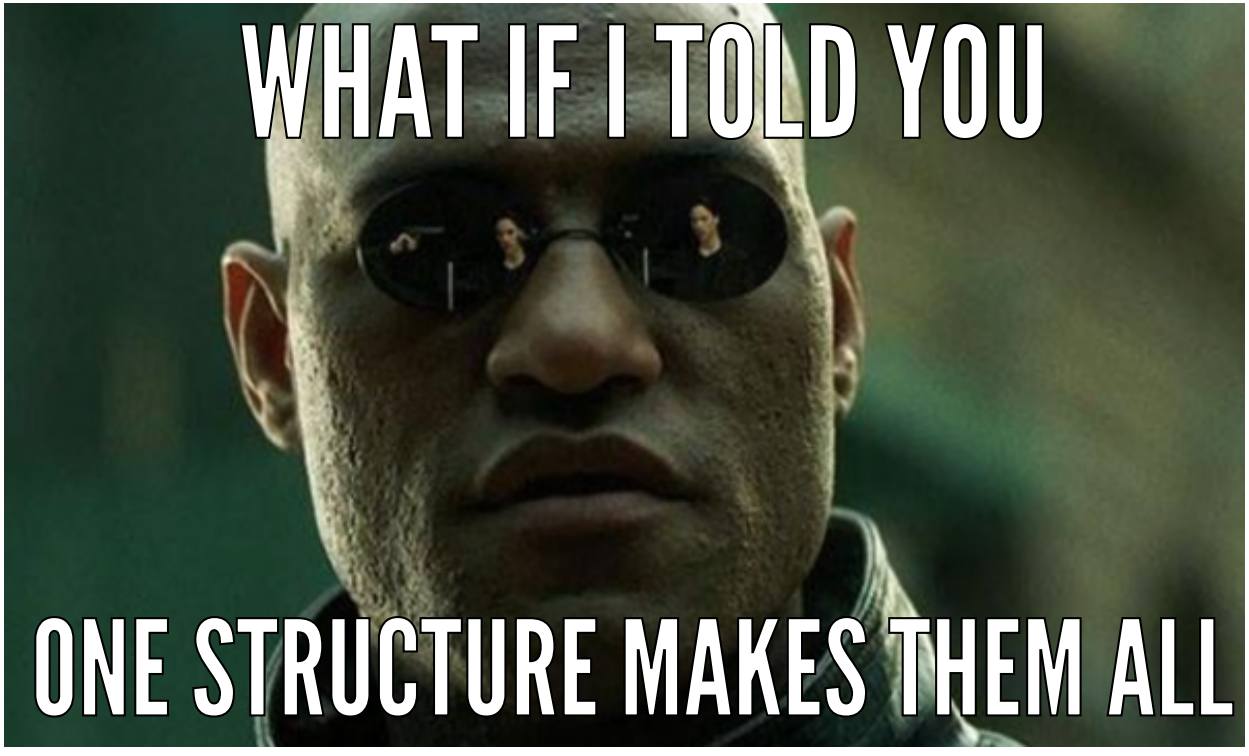

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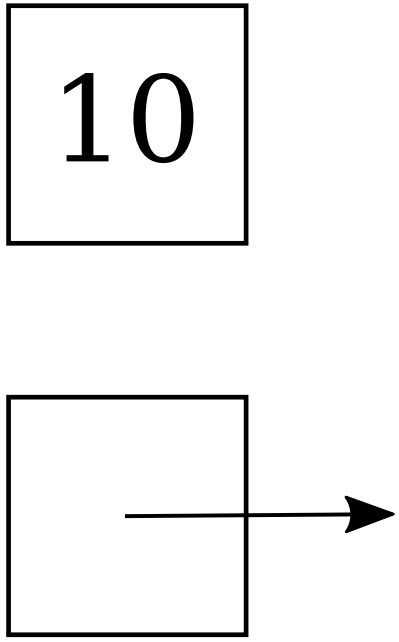
We can imagine a reference as 'pointing' to another piece of data in memory. Besides references, we also have integers and floats as primitive data (and possibly others as well).

Gettin' visual with it



We will use drawings like this one to represent data in memory.

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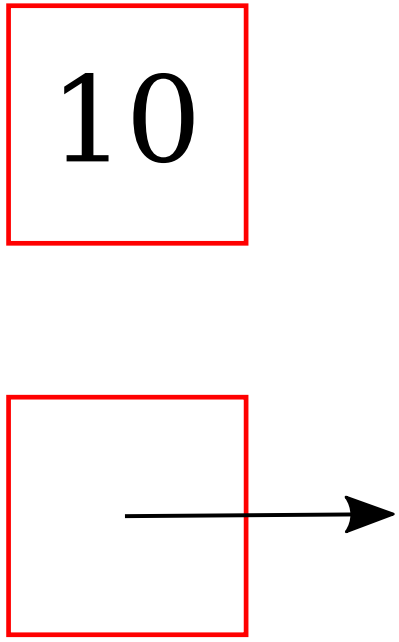

We represent machine words using boxes. Adjacent boxes are adjacent machine words in memory (these two are not).

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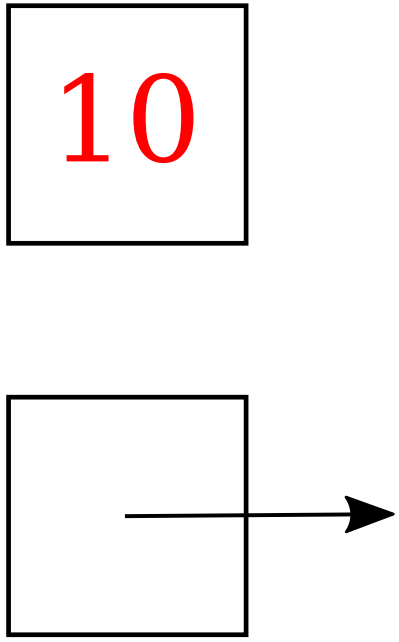

Non-reference primitive data will be drawn inside the box representing its machine word.

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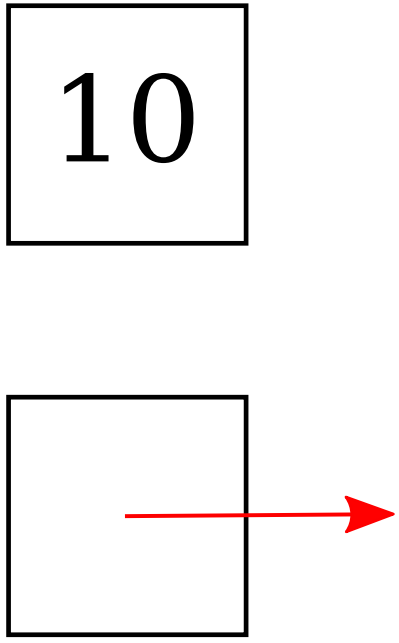
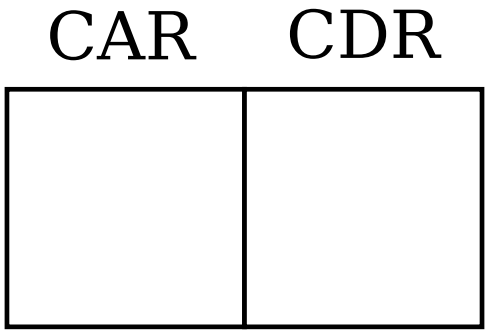


References will be drawn as arrows to the data they 'point' to (this one isn't pointing anywhere useful).

The CONS cell



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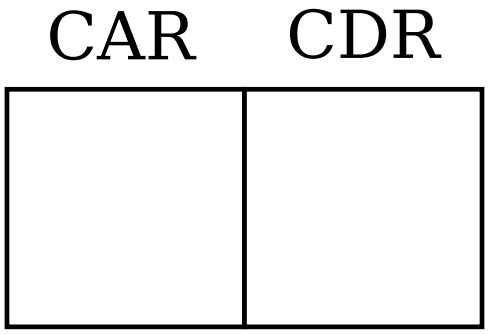

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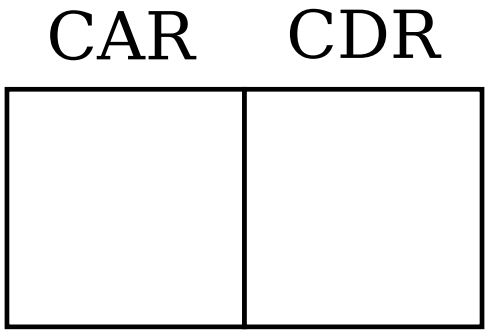

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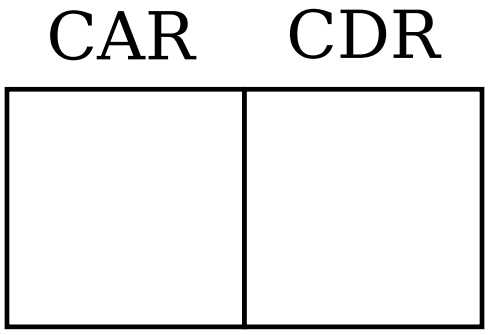

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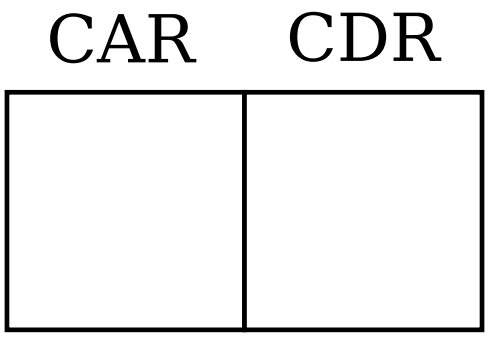

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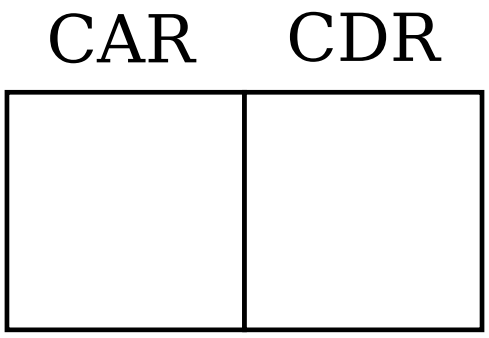

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- Either the CAR or the CDR can store reference, or primitive non-reference, data as needed
- Can implement *every single one* of the data structures mentioned at the start of this talk!

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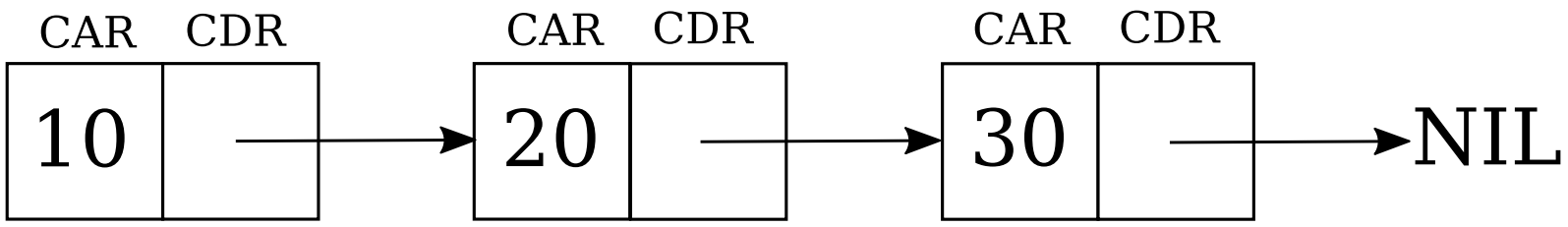
Using this approach, we can define all the usual list operations in a straightforward way.

Visualizing CONS cell lists



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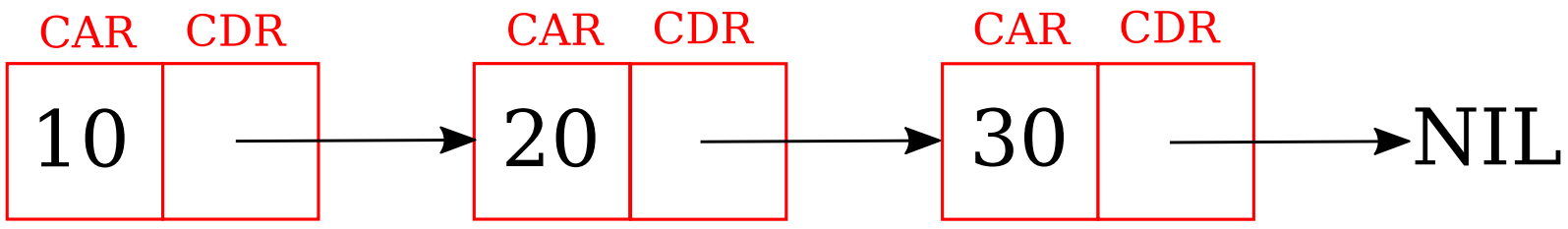

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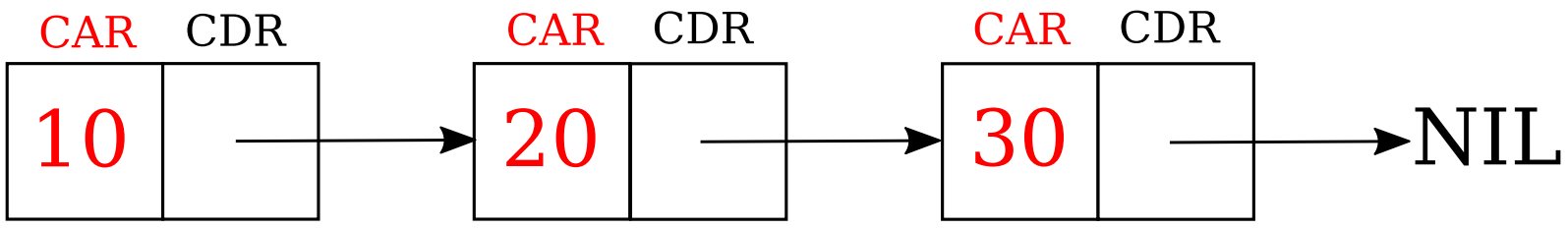

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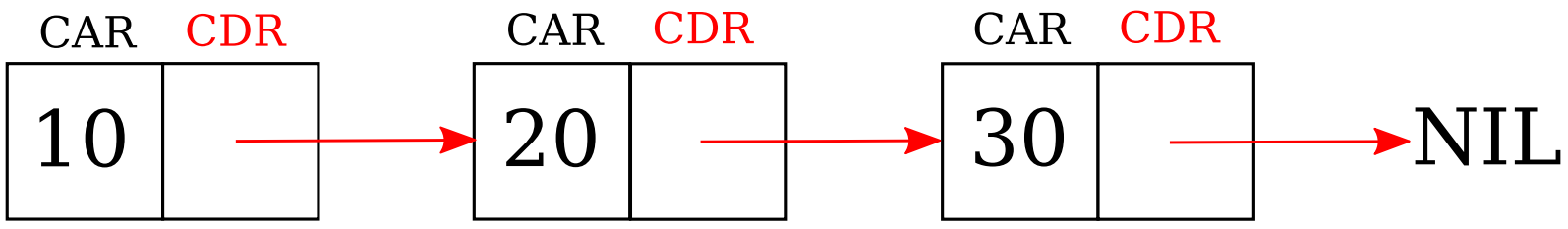

This is the in-memory representation of the list `[10, 20, 30]` using CONS cells. Each CONS cell is one list node. CARs contain the data. CDRs contain references to the next list node, or NIL for the end.

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- We can even mix-and-match the two!

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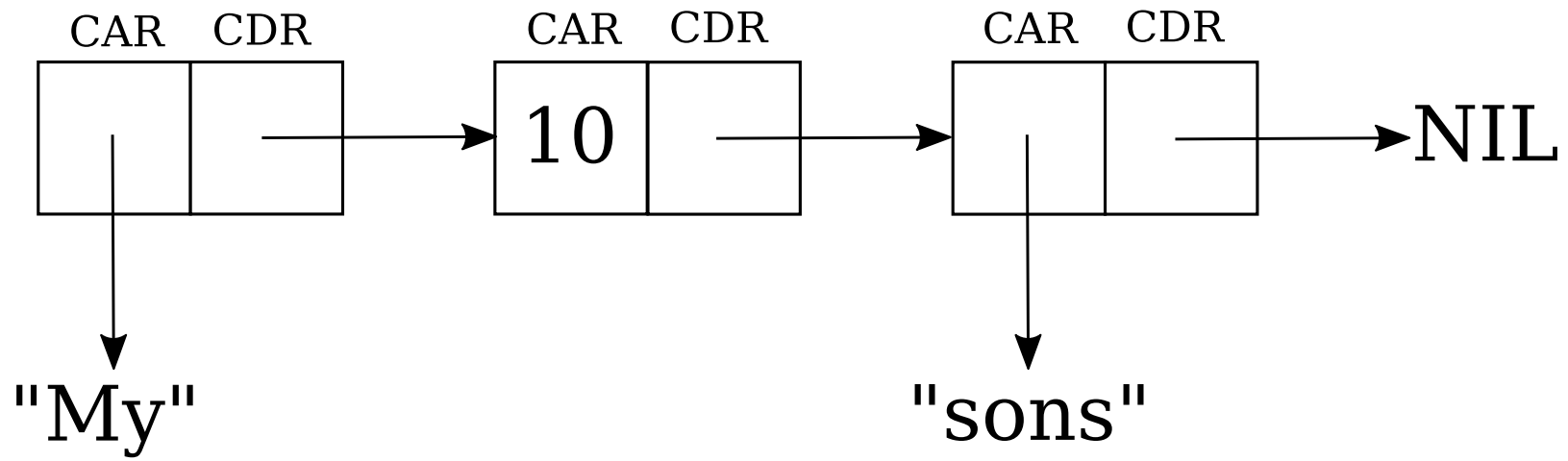

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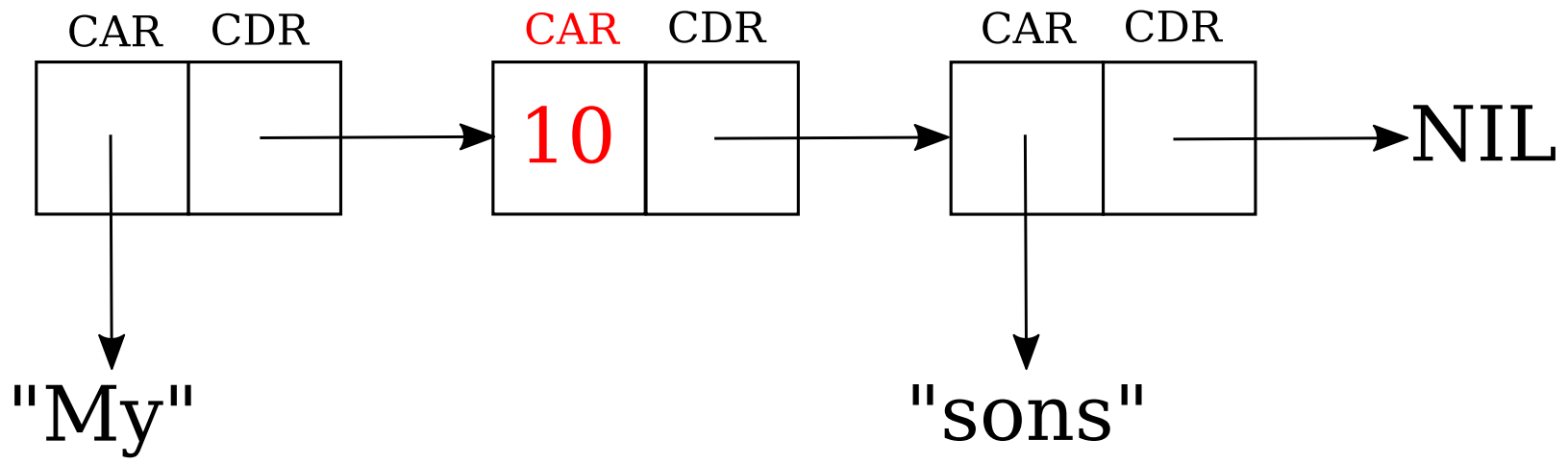

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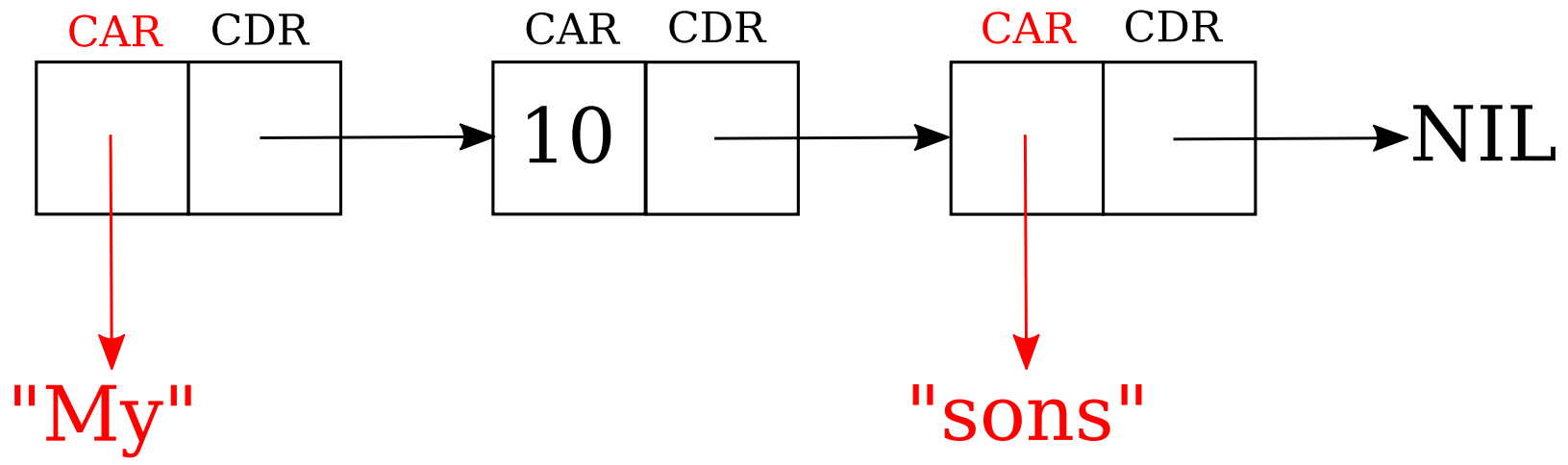

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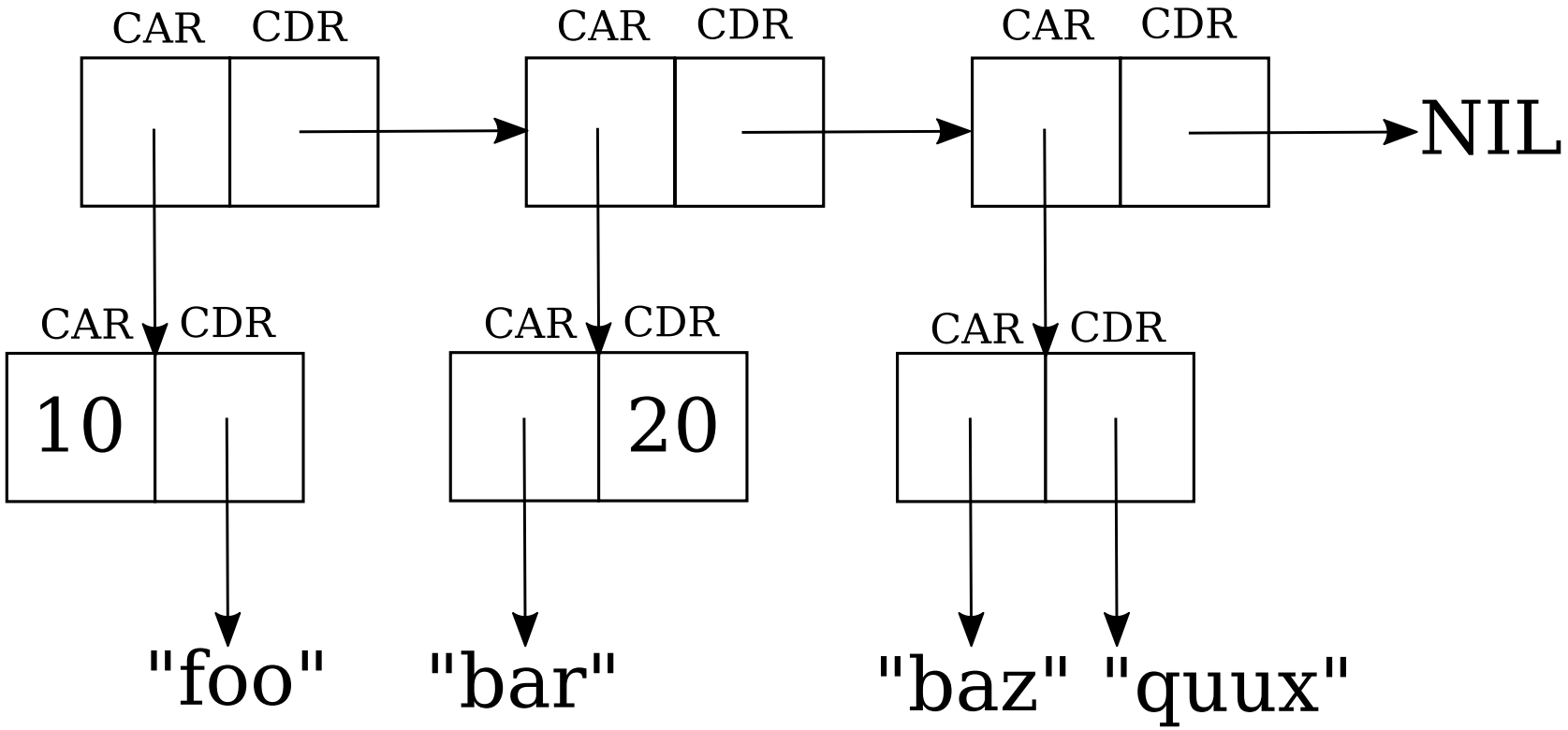
- Put inserts at the front of the list (so they'll be found before any previous entries with the same key)

Example of CONS cell-based dictionary



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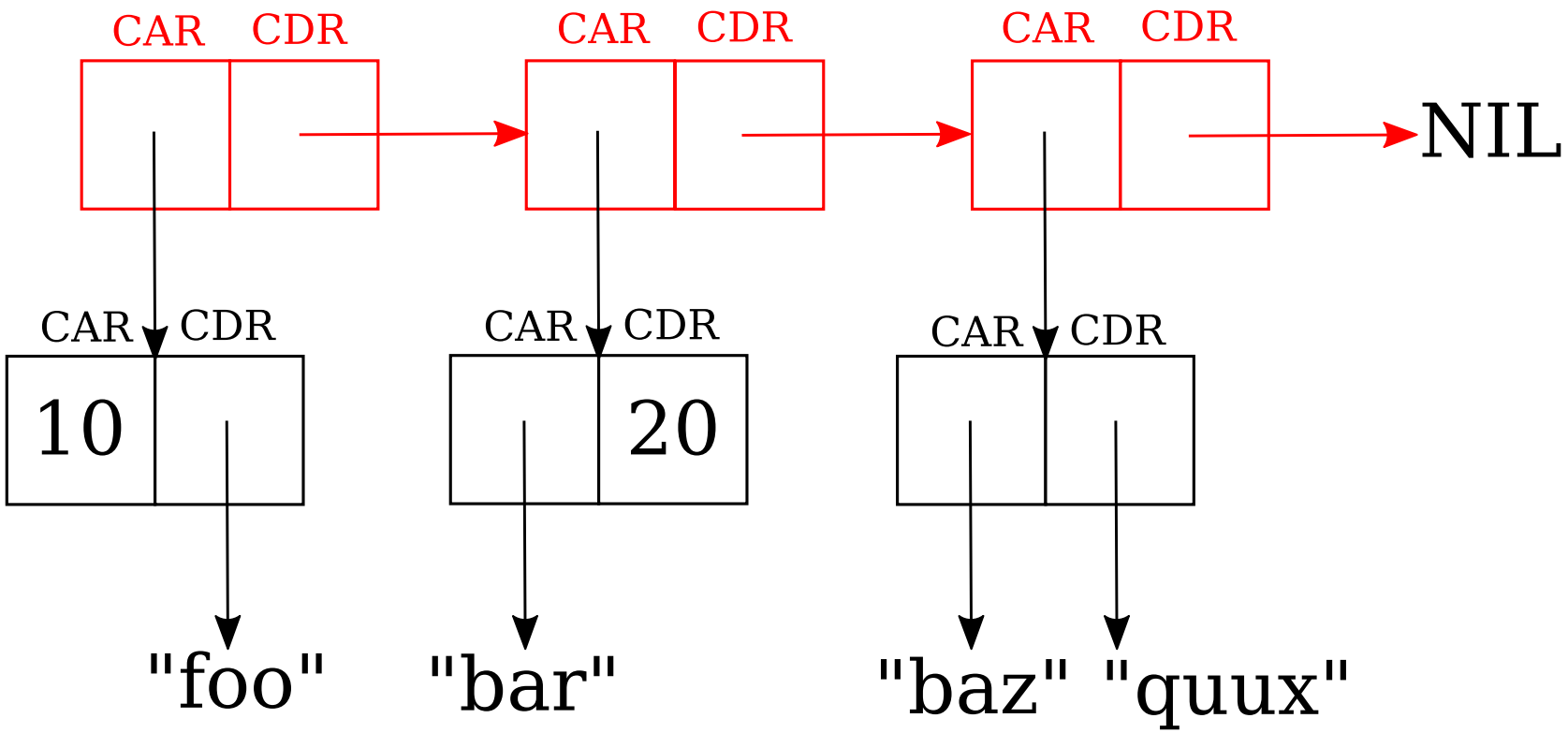

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We can define a rank n matrix (for $n > 1$) as a spine of rank $n - 1$ matrices.

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or along the rows:

```
[[1, 2, 3, 4], [5, 6, 7, 8], [9, 10, 11, 12]]
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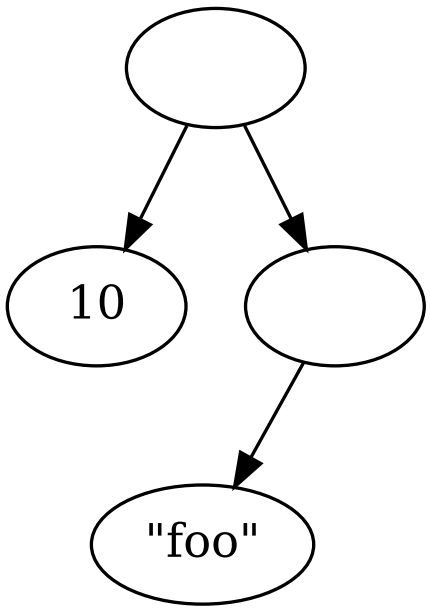
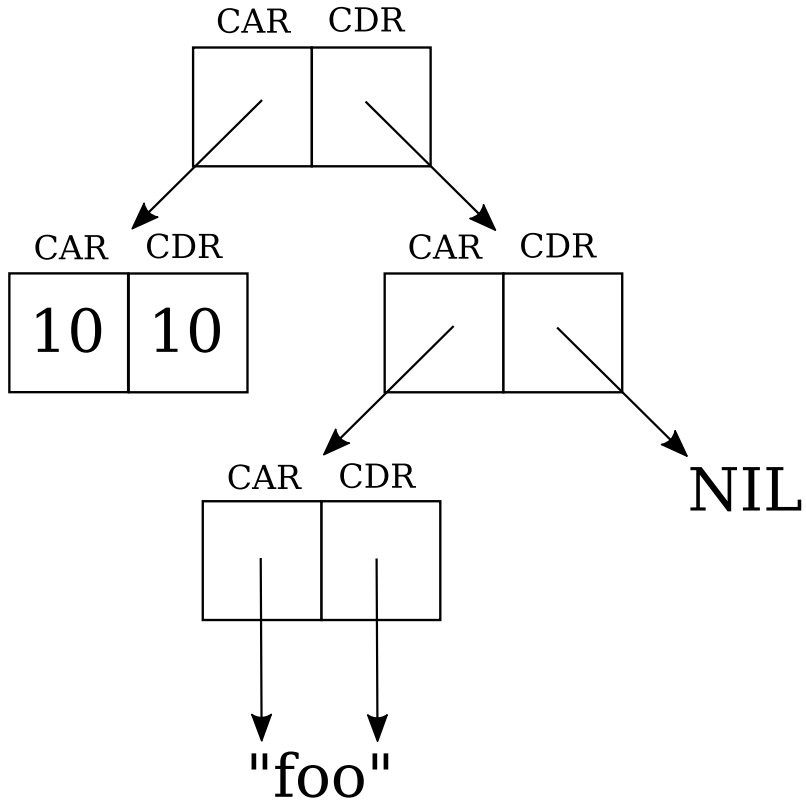
- An internal node is represented by a CONS cell whose CAR stores a reference to its left child (or NIL if none) and whose CDR stores a reference to its right child (or NIL if none)
- A leaf node is represented by a CONS cell storing the same data (or reference) in both its CAR and CDR

Example binary leaf tree and its CONS cell representation





Rose trees

If we need trees of arbitrary branching factor (number of children), or where data has to be stored in the leaves as well, we can instead use a *rose tree* representation:

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We can also include a parent reference in the list of children (typically as the first element) if we want.

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- A child list's data are references to elements of the adjacency spine, corresponding to the neighbours of that vertex

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- This means that we could potentially be getting cache misses *every time* we follow a stored reference!
- There are ways to make CONS cell-based structures more cache-friendly, but they're usually more trouble than they're worth

So why use them?

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In short, CONS cell-based structures can help you *learn your tradeoffs* quickly and easily.

Questions?