

What are algorithms,
and why do we care?

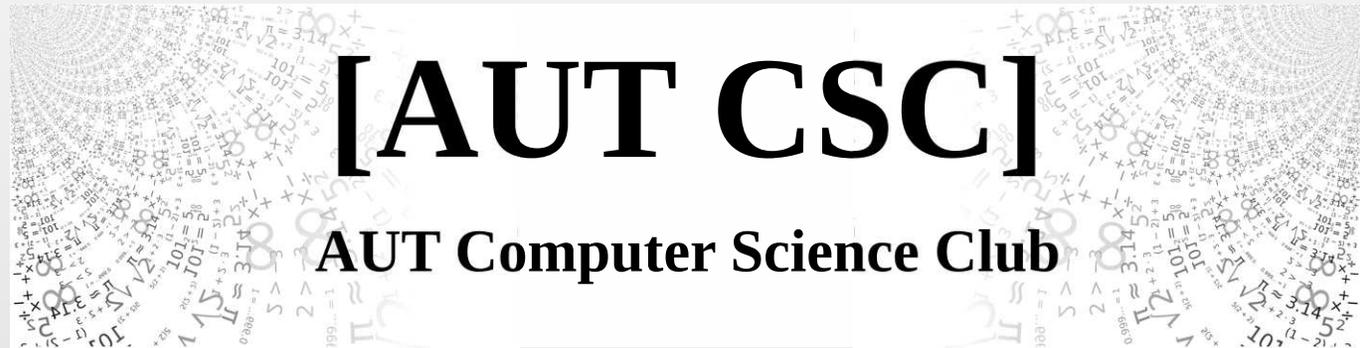
Building up

Going asymptotic

Questions

Introduction to asymptotic analysis

Or: How computer science is like cooking



Koz Ross

March 30, 2017

What are algorithms,
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Overview

What are algorithms, and why do we care?

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**What are algorithms,
and why do we care?**

- Definition of an algorithm
- Why we care
- What do we mean by 'efficient'?

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What are algorithms, and why do we care?

Definition of an algorithm

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- Why we care
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“A procedure, described in a finite number of instructions, for transforming inputs into an output to solve a specific problem. The instructions must be unambiguous, guaranteed to terminate, and solve the problem correctly in all cases.”

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Alternatively, you can think of an algorithm as a *recipe*:

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- The ingredients are the *inputs*
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- The (hopefully!) resulting food is the *output*

Why we care

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- Algorithms are *very* fundamental to computer science (which is why it's all about cooking!)

Why we care

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- Algorithms are *very* fundamental to computer science (which is why it's all about cooking!)
- They define what we can do *at all*, and what we can do *efficiently*

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Thus, we really need to be able to *compare* algorithms to each other, and also *understand* why they run the way they do, if we have any hope of making them do our bidding.

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Efficiency is ultimately doing more with less effort. We can measure the 'effort' required to run an algorithm in two ways:

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Determining these two factors is an important part of *algorithm analysis*.

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Determining these two factors is an important part of *algorithm analysis*. Additionally, any method we use must explain *why* we get the performance it claims.

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“To analyze the efficiency of an algorithm, implement it, run it on some inputs, and measure the time required and the memory used.”

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While this method is useful at the *end* of the process, it's not a very good analytical tool in general. To do better than this, we need a different view of the problem, as well as a different method.

A model

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In sciences (of all sorts), a *model* is a way of looking at the world which:

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Examples of models include:

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Examples of models include:

- The Standard Model

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Models allow us to *analyze* and *understand* their subject matter without having to 'manually verify' everything.

A model of a computer

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Because algorithms ultimately run on computers, our model must be of a computer.

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There are *many* choices of model, depending on what kind of computer we want to study, as well as what aspects of it interest us.

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The most traditional (and foundational) model for algorithm analysis is the *random access model* (RAM).

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The most traditional (and foundational) model for algorithm analysis is the *random access model* (RAM). This closely represents a computer at the time when algorithm analysis first became a topic in its own right (around 1950).

The random access model

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In RAM, we have access to:

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Any primitive instruction requires one *time unit* to execute.

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Problem: *Given a non-empty array arr of one-word integers, find the largest integer in arr.*

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Problem: *Given a non-empty array arr of one-word integers, find the largest integer in arr.*

```
function Max(arr)
  x ← arr[1]
  for i ∈ 2, 3, ..., length(arr) do
    if x < arr[i] then
      x ← arr[i]
    end if
  end for
  return x
end function
```

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Problem: *Given a non-empty array arr of one-word integers, find the largest integer in arr.*

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function Max(arr)
  x ← arr[1]
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    if x < arr[i] then
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$T_{\text{Max}}(n) =$

An example algorithm

What are algorithms,
and why do we care?

Building up

- Analysis by implementation
- A model
- Time and space complexity
- **An example algorithm**

Going asymptotic

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$$T_{\text{Max}}(n) = 4 + 6(n - 1) + 1 = 6n - 1$$

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- Asymptotic analysis
- Reviewing the example
- Benefits of asymptotic complexity

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Asymptotic analysis

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Ultimately, we don't actually care about what $T(n)$, $S(n)$ *are*.

Asymptotic analysis

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Ultimately, we don't actually care about what $T(n)$, $S(n)$ *are*. What really matters is *how they behave as their input sizes grow*. Our current method focuses too much on the former, and not enough on the latter.

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We call this kind of analysis *asymptotic*. This is because the input can grow as big as we like, and we're only interested in how the running time (or space) changes with the growth of the input.

Reviewing the example

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$$T_{\text{Max}}(n) = \cancel{6n} - 1$$

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$$T_{\text{Max}}(n) = \cancel{6n - 1} \quad \text{WRONG!}$$

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$T_{\text{Max}}(n)$ is $O(n)$

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We also say “the time complexity of Max is $O(n)$ ”, or even “Max is $O(n)$ ”.

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We also say “the time complexity of Max is $O(n)$ ”, or even “Max is $O(n)$ ”. The second one is a bit unclear, so I’d avoid it.

Benefits of asymptotic complexity

What are algorithms,
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Questions

- *Describes* performance on the basis of how our time and space requirements will grow relative the size of the problem, not pegged to a particular machine (or even model!)

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- *Describes* performance on the basis of how our time and space requirements will grow relative the size of the problem, not pegged to a particular machine (or even model!)
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It’s definitely not perfect though — but that’s material for another talk.

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