Sorting

Or: some of what Koz spent about two years of his life on

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[AUT CSC]

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Outline

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As a result, sorting is one of *the* oldest computer science problems we have, and has been studied *to death*.

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Without further ado, let's commence!

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$$\{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$$

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We denote the special case of $A \times A$ as A^2.

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We can think of a relation as explicitly spelling out what things in A and B are related.

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We can read $a \leqq b$ as 'a does not come after b according to the ordering $\leqq$'. We can have multiple orderings on various sets.

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We can also do something similar with $\geq$. If we look at other sets, we get many other orderings (e.g. lex and reverse lex for strings).

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Put simply, the sorting problem requires us to put t 'in order' according to $\leqq$.

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- Closest to a 'pure' view of how hard the sorting problem is (no 'extra baggage' to confuse analysis)

Our approach to analysis

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- Tuple elements are 'random' (i.e. no sorted sub-sequences)

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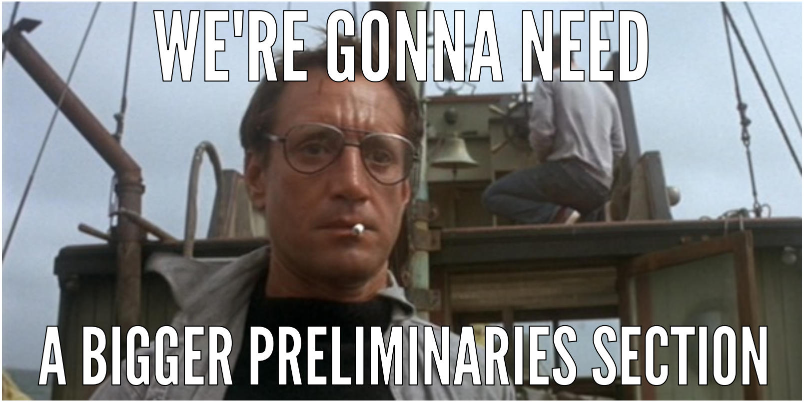
WE'RE GONNA NEED
A BIGGER PRELIMINARIES SECTION

Factorials and permutations

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Let $n \in \mathbb{N}$. The *factorial of* n is

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Example

Two possible permutations of $S = \{1, 2, 3\}$ are $(1, 3, 2), (2, 3, 1)$.

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When $n > 0$, suppose that the lemma holds to some $k \geq 0$. Thus, there are $k!$ permutations of any k-element set S. Without loss of generality, we observe that any $k + 1$-element set S' is equal to some k-element set S with an additional element $u \notin S$. Thus, to convert a permutation of S into a permutation p of S', we need to 'insert' u into p. As there are $k + 1$ possible positions to insert into for each permutation of S, the number of possible permutations of S' is thus $k!(k + 1) = (k + 1)!$. Thus, the lemma holds for all n. $\square$

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Corollary

Under our assumptions, no comparison sort with a time complexity better than $\Theta(n \log(n))$ (and thus, $O(n \log(n))$) can exist.

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- RAM is not the most accurate model of modern computers:
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  - Modern memory is *very* hierarchical — also lots of optimality points to consider

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- We usually know more about our data (numbers, limited number of unique items, strings, etc)
- RAM is not the most accurate model of modern computers:
  - Modern machines are parallel — lots of different optimality points there!
  - Modern memory is *very* hierarchical — also lots of optimality points to consider
- Data is not static or centralized anymore:

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Still work to be done in this area — for many years to come!

Questions?